

February 2, 2008

Water temperature constraint on Sonoluminescence

Sohrab Rahvar ¹

Atomic Energy Organization of Iran, Bonab Research Center, P.O.Box 11365–8486,
Tehran, Iran

Institute for Studies in Theoretical Physics and Mathematics, P.O.Box 19395–5531,
Tehran, Iran

Abstract

It is proposed that shock wave dynamics within the gas of a small bubble explain sonoluminescence, the emission of visible radiation. As the bubble radius oscillates, shock waves develop from spherical sound waves created inside the gas bubble. As any such shock propagates toward the center, it strengthens and, upon convergence and subsequent reflection, temperature of gas inside bubble increases dramatically in such a way that it can produce plasma. Since main radiation product in exploding epoch, nonadiabatic condition for imploding shock wave cool plasma and cause exploding shock wave can not sufficiently rise temperature to produce radiation. In this work we compare cooling time for plasma by bremsstrahlung radiation with collapsing time for the imploding shock wave. We find a constraint on radius of bubble with respect to temperature of water. This constraint condition explains experimental results as to, why the cold water is fine for SL.

Keywords: Sonoluminescence–Bremsstrahlung Radiation–Shock Wave

¹e-mail:rahvar@physics.sharif.ac.ir

1 Introduction

Sonoluminescence, the phenomenon of emission of light from small bubble occurring during ultrasonic excitation, has been known for more than half a century. In this phenomenon an intense standing wave increases the pulsation of a bubble of gas trapped at a velocity node to attain sufficient amplitude so as to emit picosecond flash of light. The analysis of the dynamics of a small bubble or cavity in a fluid dates back to the work of Lord Rayleigh [1] at the beginning of this century. A large number of publications followed in subsequent decades, including the studies of oscillating bubbles by Plesset [2], Eller & Crum, Flynn[3], Lauterborn[4], Prosperetti[5], and many others. Experiments indicate that the collapse is remarkably spherical, and that water is the best fluid for SL. Some noble gas is essential for stable SL, and that the light intensity increases as ambient temperature is lowered. The theory of light emitting mechanism is still open, but the traditional scenario is supersonic bubble collapse launching an imploding shock wave which ionizes the bubble contains so as cause it to emit. Bremsstrahlung radiation is the best theory for SL phenomenon. In the theory of radiation by Bremsstrahlung as the bubble collapses, produced shock wave ionized the gas and cause it to radiate. The imploding shock wave concentrated at the center of bubble, through this concentration, shock wave warmed the gas until will receive to the center of bubble and explode, through this explosion gas which was compressed just behind the imploding shock front now finds itself in front of the shock front again. As the shock passes through these particles a second time, there is another burst of heating and maximum temperature reached by the exploding shock wave [6]. The mean idea of our work is that, if in the imploding regime the plasma cools by radiation then in the exploding epoch, shock front can not warm particles again to produce intensive radiation. Thus by comparing cooling time with dynamical time for collapsing, we can say that if cooling time is less than collapsing time then plasma inside bubble will cool and the exploding shock wave can not warm this gas again. This constraint gives us some useful parameter for SL, therefore, we can explain results of some experiments such as why the light intensity increases as ambient temperature is lowered[7].

2 Collapse Mechanism and radiation

Explanation of the light-emitting mechanism of SL naturally seeks to interpret the featureless spectrum in terms of emission from a hot spot, for example black body radiation, if the radiation and matter are near to equilibrium, or Bremsstrahlung from accelerating unbound electrons if the light-emitting region is hot enough to be ionized yet sufficiently rarefied so as to be transparent to radiation[8]. The shock wave model [9,10,6,11] provides an extra stage of energy focusing by assuming that supersonic inward collapse of the bubble wall launches a shock wave into the bubble's interior. This shock can run through the already compressed gas inside the bubble, increasing its amplitude and speed as it focuses towards the origin. There is now a surface of radius R_s (the radius of the shock front), within the bubble of radius R . The similarity solution is obtained by assuming that the shock radius takes this form :

$$R_s = A(-t)^\alpha, \quad (1)$$

(Guderley 1942) [12] where time is measured from the time of the convergence of the shock, and A is the "launch" condition of the shock, which couples the shock to the bubble motion. The similarity solution yields an exponent α of 0.72 in air and 0.69 for noble gases. Since the exponent is less than unity, the Mach number of shock as goes to the origin approaches infinity. The bubble wall is collapsing at the speed of sound when it is passing through its ambient radius. In this case (Barber et al)[6] showed that:

$$R_s = R_0 \left(-\frac{t}{t_0}\right)^\alpha, \quad (2)$$

where

$$t_0 = \alpha \frac{R_0}{c_0}, \quad (3)$$

In terms of Mach number

$$M = \left| \frac{t_0}{t} \right|^{1-\alpha}. \quad (4)$$

one can define dynamical time or collapsing time according to:

$$t_{dy} = \frac{R(t)}{\frac{dR(t)}{dt}}, \quad (5)$$

$$t_{dy} = \frac{\alpha R_0}{c_0} \left(\frac{1}{M}\right)^{\frac{1}{1-\alpha}}. \quad (6)$$

where t_{dy} is the time scale for the shock to have a radius smaller than R_s . As the shock wave approaches the center the temperature immediately behind the imploding shock front increases by this factor $\frac{T}{T_0} = M^2$. When the shock wave converges to the origin it explodes from the origin with the same similarity solution. Thus the gas which was compressed just behind imploding shock front now finds itself in the front of the shock front again. As the shock passes through these particles a second time, there is another burst of heating and the maximum temperatures reached by the exploding shock wave now increases by the factor $\frac{T}{T_0} = M^4$ [7].

3 Cooling Condition for Radiation

The temperature increase inside the bubble by shock wave ionizes the region. The free electrons released by the heating will accelerate and radiate light as they collide with the ions. The Bremsstrahlung radiation generated per second per volume is equal to :

$$\frac{dE}{dt dv} = \frac{16z^2 e^6 n^2}{3m^2 c^3 h} \left(\frac{2mkT}{\pi} \right)^{1/2} g_B \approx 1.5 \times 10^{-27} n^2 T^{1/2} g_B. \quad (7)$$

where e and m are the electron charge and mass, n the density of free electrons and ions, c the speed of light, and g_B the Gount factor. In the temperature of T , the energy density of this plasma is equal to:

$$\frac{dE}{dv} = \frac{3}{2} nkT. \quad (8)$$

So we can introduce cooling time for this plasma

$$t_{cool} = \frac{\frac{dE}{dv}}{\frac{dE}{dt dv}}. \quad (9)$$

$$t_{cool} = \frac{9m^2 c^3 h}{32z^2 e^6} \left(\frac{\pi}{3mk} \right)^{1/2} n^{-1} T^{1/2}. \quad (10)$$

$$\approx \frac{0.26}{g_B} \times 10^{12} n^{-1} T^{1/2}. \quad (11)$$

As the bubble collapses the value of n changes with the radius, by using the dynamical equation of shock wave we can interpret the above density in terms of initial density and radius of gas in the bubble as:

$$n = n_0 \left(\frac{R_0}{R_s} \right)^3. \quad (12)$$

By using dynamical equation for shock wave, cooling time scale can be obtained in terms of Mach number

$$t_{cool} = \frac{0.26}{g_B} \frac{10^{12}}{z^2 n_0} \left(\frac{1}{M} \right)^{\frac{3\alpha}{1-\alpha}} T^{1/2}. \quad (13)$$

Now we compare this time scale with dynamical time scale for the shock wave. If $t_{cool} > t_{dy}$ then for the imploding shock wave, as the shock wave implodes toward the origin, plasma cannot cool thus adiabatic condition holds. So in the explosion epoch temperature can rise sufficiently to produce an extensive radiation. Now if $t_{cool} < t_{dy}$, then as the imploding shock wave implodes towards the center of bubble by the radiation mechanism, plasma cools and temperature can not rise in such a way that exploding shock can produce an intensive radiation. As mentioned before, temperature behind the shock wave rises by a factor of $\frac{T}{T_R} = M^2$, where $T(R)$ is the temperature of the bubble when its radius is R . For the constraint $t_{cool} > t_{dyn}$, temperature of gas inside the bubble rises with an adiabatic compressing:

$$T(R) = T_0 \left(\frac{R_0}{R} \right)^{3(\gamma-1)}, \quad (14)$$

where T_0 is the ambient temperature of bubble and R_0 is the size of the bubble when it is in a static mechanical equilibrium. By using shock wave dynamics, the above equation can be obtained in terms of Mach number, so we have:

$$T(R) = T_0 M^{\frac{3}{2} \frac{\alpha}{1-\alpha} (\gamma-1)}. \quad (15)$$

By substituting Eqs.(6 & 13) in the adiabatic condition for imploding shock wave, constraint condition obtain in the term of ambient temperature, Mach number, initial density and initial radius and constants of gas

$$R_0 < \frac{0.26}{g_B} \frac{c_0 10^{12}}{\alpha z n_0} M^{\frac{3\alpha\gamma-11\alpha+4}{2(1-\alpha)}} T_0^{1/2}, \quad (16)$$

where c_0 is the velocity of sound and n_0 is the initial density of gas inside the bubble. From experiment we have $n_0 \approx 4.16 \times 10^{19} \frac{par}{cm^3}$ and $g_B = 1.2$. Measurements indicate that the bubble wall is collapsing at more than 4 times the ambient speed of sound in air[13]. So the above inequality can be written this way:

$$R_0 < 0.258 \times 10^{-4} T_0^{1/2}. \quad (17)$$

As we said temperature rises by two processes:

- i. By shock wave $T = M^2 T(R)$.

ii. By adiabatic collapsing of bubble $T(R) = M^{(\gamma-1)(\frac{3\alpha}{1-\alpha})}T_0$.

The factor of increase in temperature in the above processes is $T = M^2 M^{(\gamma-1)(\frac{3\alpha}{1-\alpha})}T_0$. For air, we have $T = 1144 \times T_0$ where T_0 is the ambient temperature, If we consider room temperature for ambient temperature, the temperature of bubble rises up to 10^5 . In the Bremsstrahlung radiation we considered all of our gases to be ionized. Verification of that statement can be obtained by Saha's equation

$$\frac{q^2}{1-q} = 2.4 \times 10^{21} T^{3/2} \exp(-\frac{\chi}{kT}) \frac{1}{n} \quad (18)$$

where $q = \frac{n_e}{n}$ is the degree of ionization, χ is the ionization potential, and the pre factor T is given in Kelvin. If we put the given parameters for SL in Saha's equation for air, we find that $n_e = n_i = n$. This is the reason for considering completely ionized gas Bremsstrahlung formula. The inequality curve of Eq.(17) gives us a constraint for producing radiation in SL (Fig). On the other hand Bradley et al (1994) show the extreme sensitivity of SL to external parameters such as the water temperature. They show that as the water temperature decreases from 40° to 1° , the intensity of light emission increases by a factor of over 200. At about 0° the purple light emitted by the bubble is so bright that one can see it by an unaided eye, but at 40° the SL is barely visible even in a darkened room. According to Bradley et al (1994) by light scattering technique initial radius of bubble obtained respect to ambient temperature. Comparing constraint curve for SL with the experimental relation between ambient radius of bubble and temperature (Fig), we see that above about 25° constraint for SL breaks down and confirms the experiment results of Bradley et al (1994).

4 Acknowledgment

I would like to thank Dr.Rasool Sadighi and Dr.Kamal Seied yaghobi for usefull discussion and comments.

References

- [1] Lord Rayleigh, Philos. Mag. 34, 97(1917).
- [2] Plesset, M, J. Appl. Mech. 16, 277(1949).
- [3] Eller, A. and H. G. Flynn, 1965, J. Acoust. Soc. Am. 37, 493.
- [4] Lauterborn, W. and H. Bolle, 1975, J. Fluid Mech. 72, 391.
- [5] Prosperetti, A., 1977, Quart. Appl. Math. 34, 339.
- [6] Barber, B. P., C. C. Wu, R. Lofstedt, P. H. Roberts, and S. J. Putterman, 1994, Phys. Rev. Lett. 72, 5276.
- [7] Bradley P. Barber, C. C. Wu, Ritva Lofstedt, Paul H. Roberts, and S. J. Putterman, 1994, Phys. Rev. Lett. 72, 1380.
- [8] Wu, C. C and P. H. Roberts, 1993, Phys. Rev. Lett. 70, 3424.
- [9] Jarman, P., 1960, J. Acoust. Soc. Am. 32, 1459.
- [10] Greenspan, H. P. and A. Nadim, 1993, Phys. Fluids A 5, 1065.
- [11] Moss, W. C., J. W. White, R. A. Day, and D. B. Clarke, 1994, Phys. Fluids 6, 2979.
- [12] Guderley, G., 1942, Luftfahrtforschung 19, 302.

5 Figure Caption

Bar lines indicated the experimental measurement between ambient temperature and initial radius of bubble. On the other hand there is a constraint line form Eq.(17), in such a way that below this line flashes of light can produce in SL. About 25° experimental bars cross our line and causes to SL break downs. This confirms the results of Bradley et al (1994)